UNIVERSITY OF CAMBRIDGE

Gaussian Processes for Mitigating Systematics

A Bayesian Method to Mitigate the Effects of Unmodelled Time-Varying Systematics for 21-cm Cosmology Experiments

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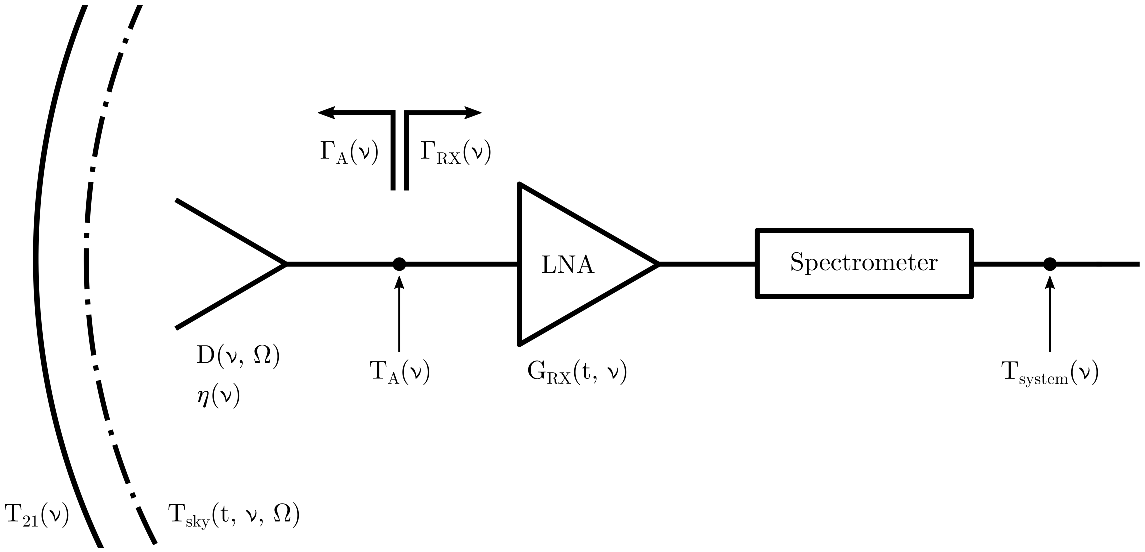
Introduction

- Origins of time-varying systematics
- Systematic model
- Incorporating Gaussian Processes into the REACH pipeline
- Results
- Further work
    - GP Regression
    - Automatic Kernel Selection

Time Varying Systematics in the REACH System

- Many sources of systematic error in the REACH system
- Some are expected to be static, others vary with time
  - Reflections from the soil vary with rainfall (Bevins et al. 2021, 2022b)
  - Impedences in the system are temperature dependent
  - Improperly modelled beams can cause systematics which vary with the galactic foreground



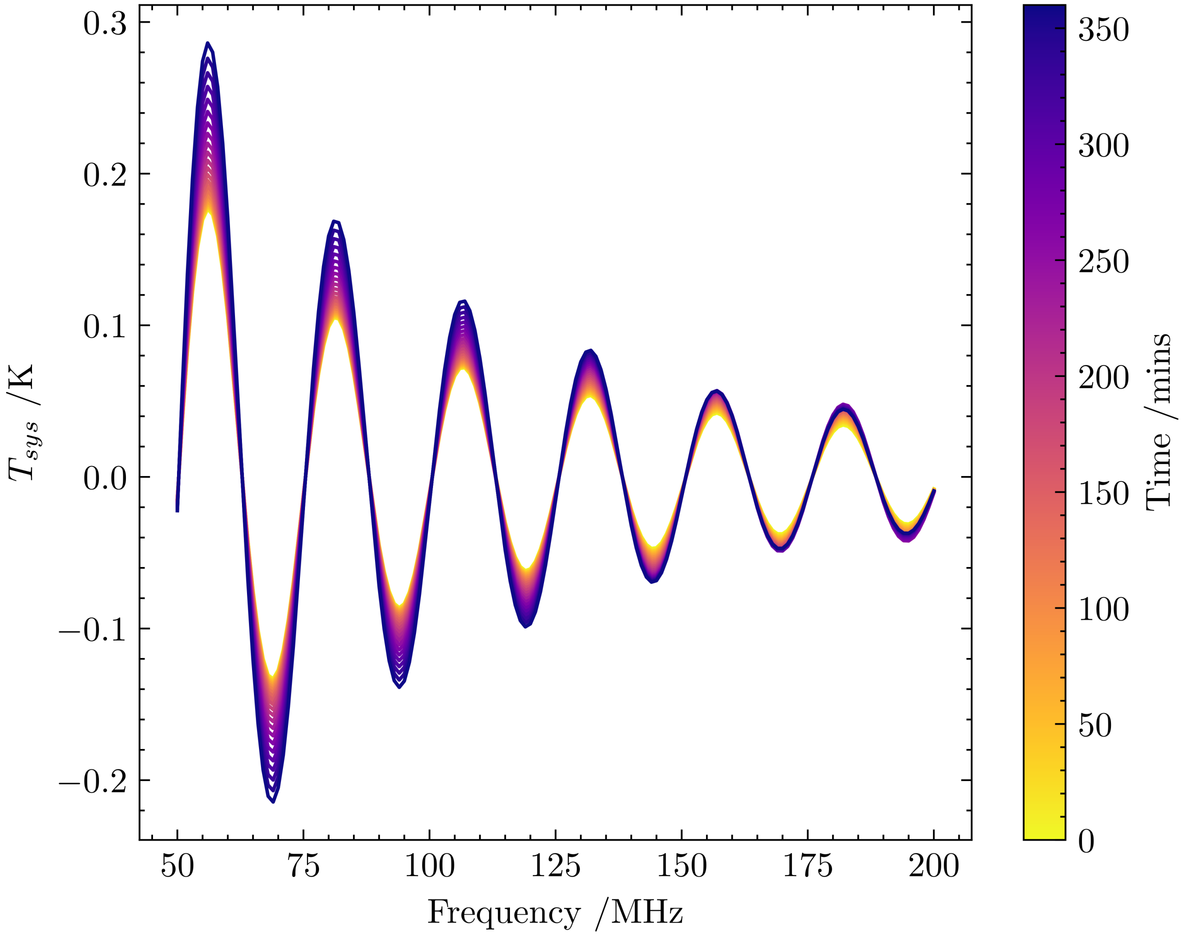
Modelling Time Varying Systematics

- Model systematic as a sinusoid whose amplitude is modulated by the power of the galactic foreground

$$T_{\rm sys}(\nu) = A_{\rm sys}\left(\frac{\nu}{\nu_{0,\rm sys}}\right)^{-\alpha_{\rm sys}} \sin\left(\frac{2\pi\nu}{P_{\rm sys}} + \phi_{\rm sys}\right)$$

- Systematic amplitude is varied like

$$A_{\rm sys}(t_j) = A_{\rm sys}(t_0) \cdot \frac{T_{\rm fg}(\nu = \nu_0, t = t_j)}{T_{\rm fg}(\nu = \nu_0, t = t_0)}$$



Introducing Gaussian Processes

- Define a GP as a collection of random variables which have consistent joint Gaussian distributions (Rasmussen 2004)
- Distribution is defined by the covariance matrix, or covariance function

$$\mathbf{C}_{ij} = K(t_i, t_j)$$

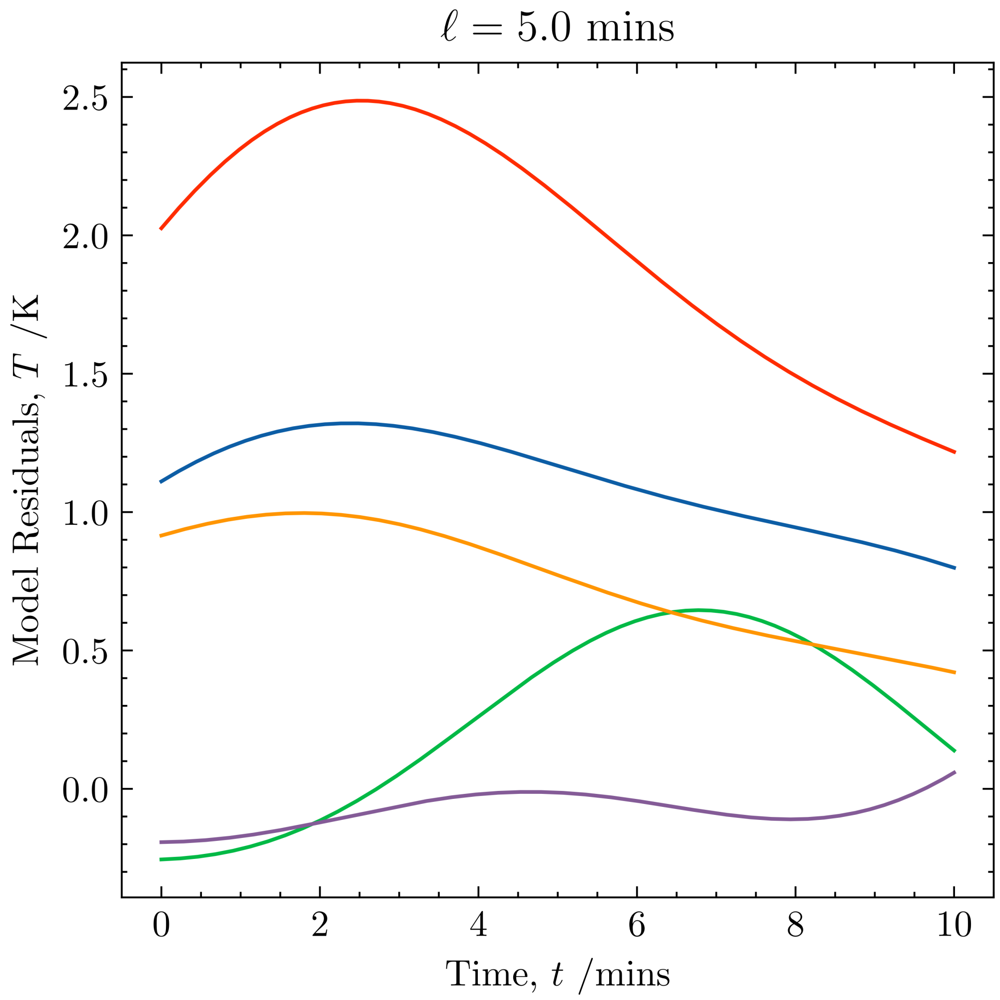
- K is called the “kernel” – form can be chosen freely

Squared Exponential Kernel

- In this work I use a squared exponential kernel with a white noise term

$$\mathbf{C}_{ij} = \sigma^2_{0,\mathrm{GP}} + \sigma^2_{\mathrm{SE}} \exp\left(-\frac{|t_i - t_j|^2}{2\ell^2}\right)$$

- We have uncorrelated white noise term, scale factor of the SE kernel and the covariance length – "hyperparameters"
- Define smooth functions



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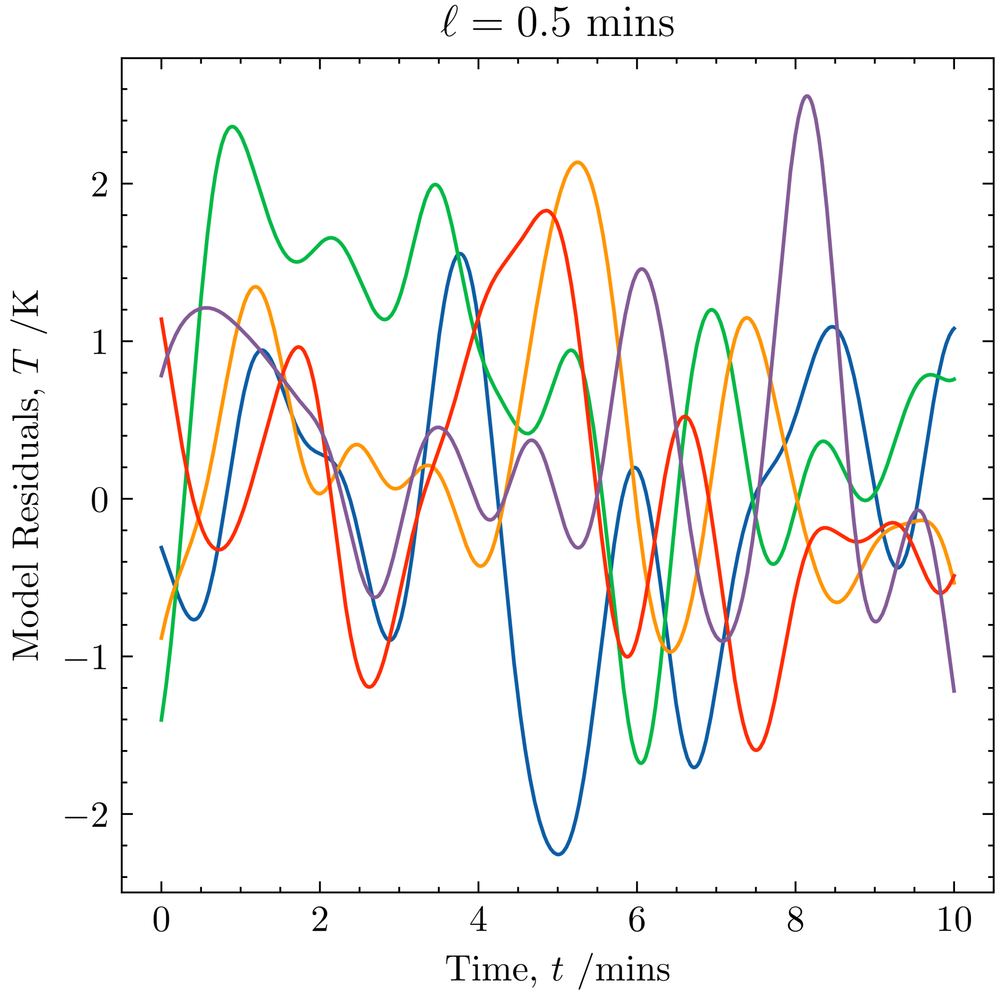


Gaussian Processes in the REACH Pipeline

- Hereafter called "Gaussian Process pipeline"

$$\mathcal{L}(\theta) = \frac{1}{\sqrt{(2\pi)^n |\mathbf{C}|}} \exp\left(-\frac{1}{2}(\mathbf{D} - \mathbf{M}(\theta))^T \boldsymbol{C}^{-1}(\mathbf{D} - \mathbf{M}(\theta))\right)$$

- **D** is the data, **M** is the model (foreground, beam model, Gaussian signal model)
- Gaussian process covariance matrix, **C**, fits covariance of model residuals between time bins
- "Standard pipeline" (Anstey et al. 2021, 2022) – uncorrelated time bins

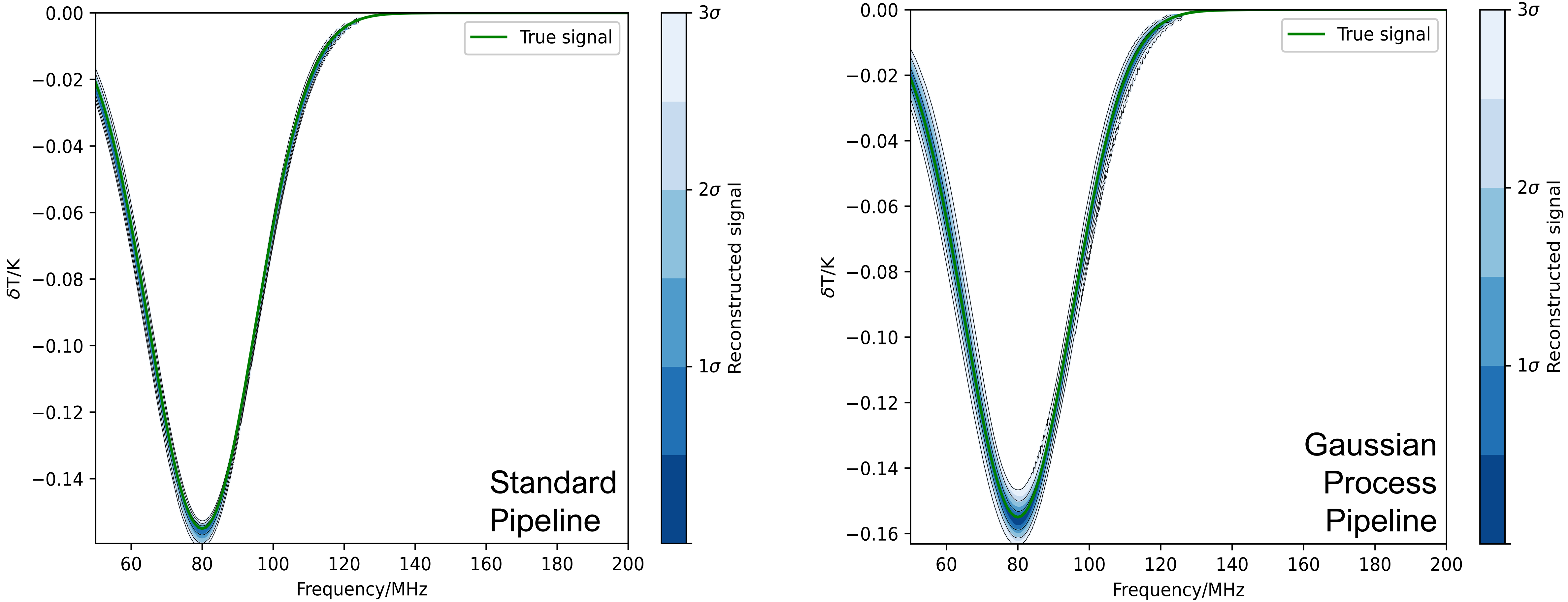
Results

- In all cases added a Gaussian signal model into the data and fit for Gaussian

$$A_{21} = 0.155\,K$$
$$\sigma_{21} = 15\,MHz$$
$$\nu_{0,21} = 85\,MHz$$

- Parameters of systematic varied relative to signal parameters
- Data generated for 24 time bins of length 15 minutes – 6 hours of observation in total

Results – Example Comparison of Signal Posteriors – No Systematic



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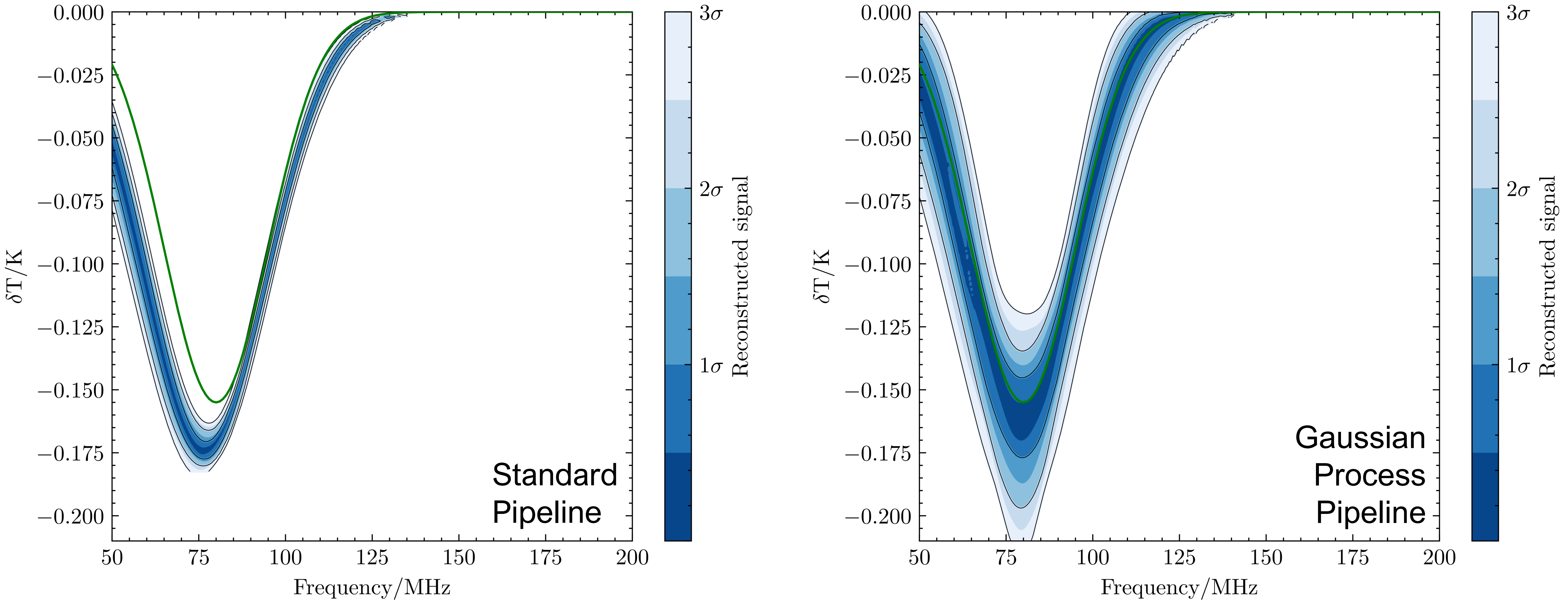
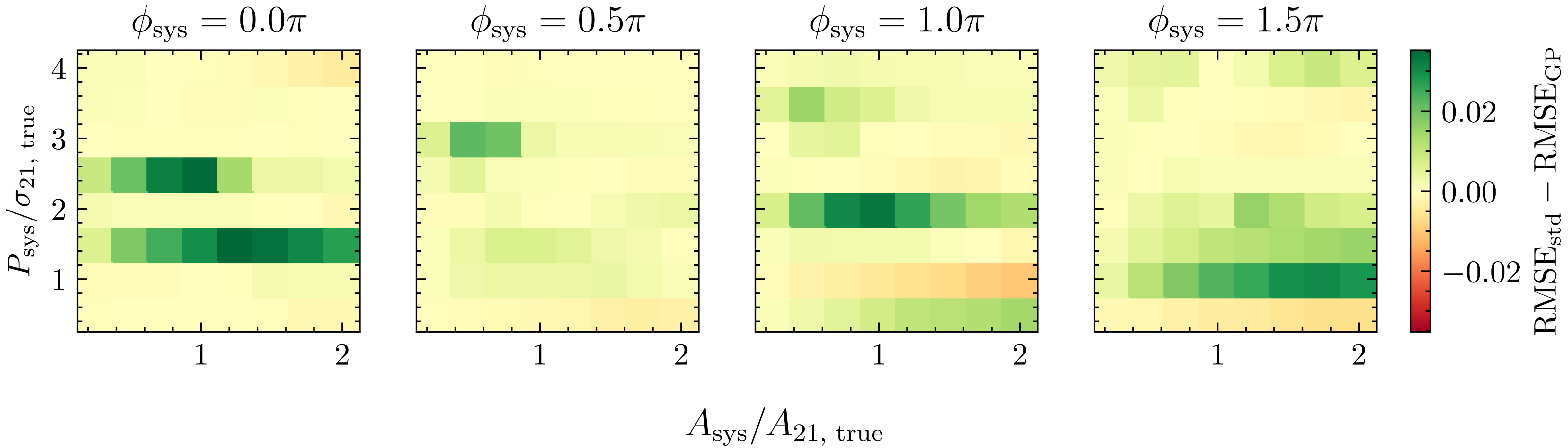
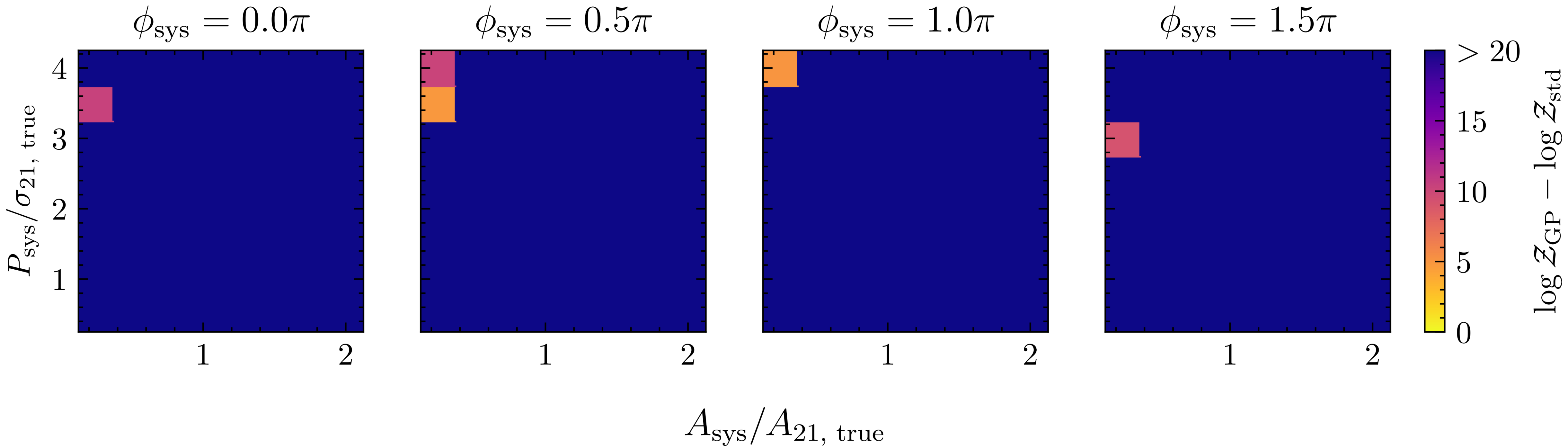


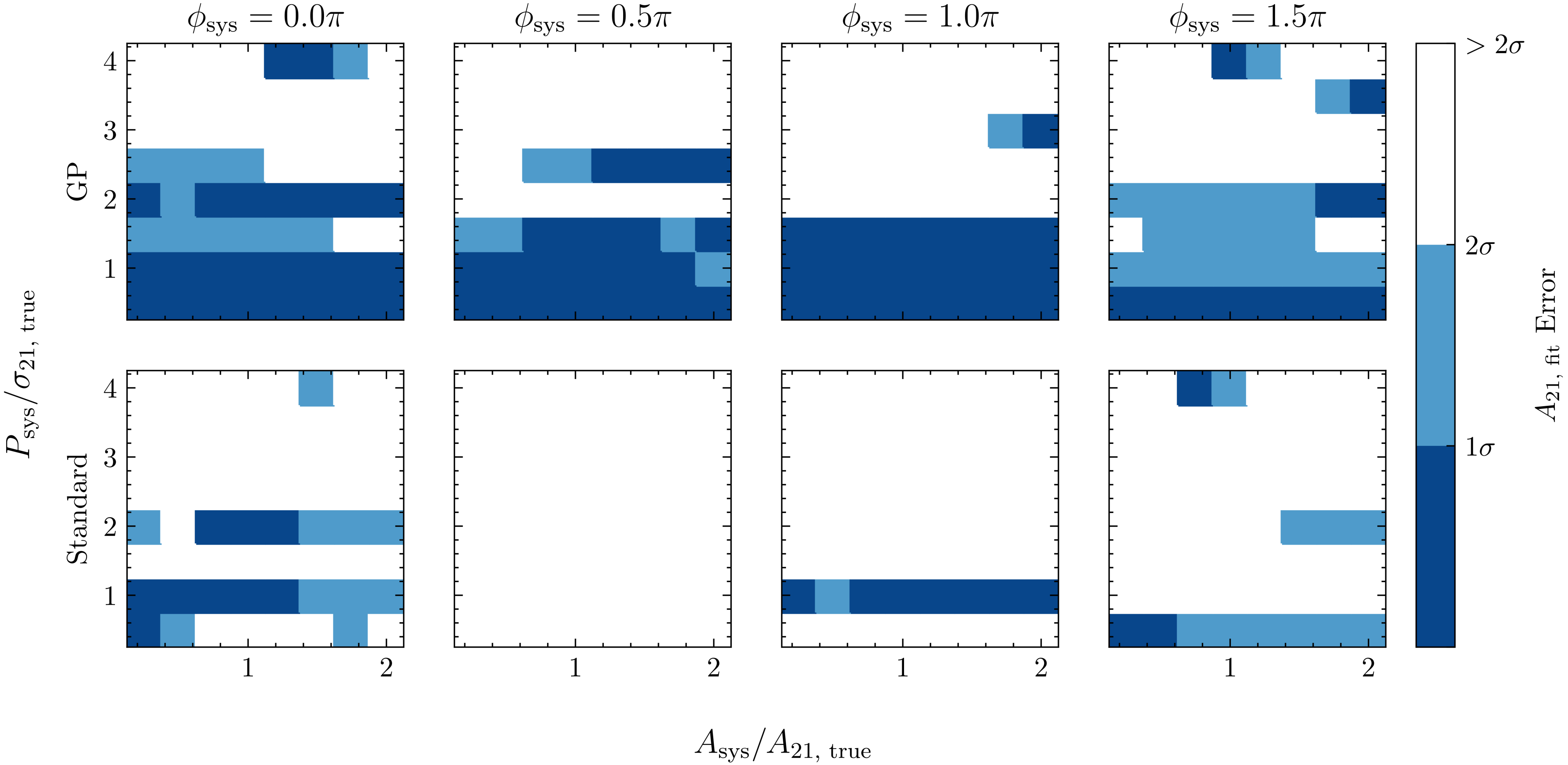
Results – Weighted RMSE



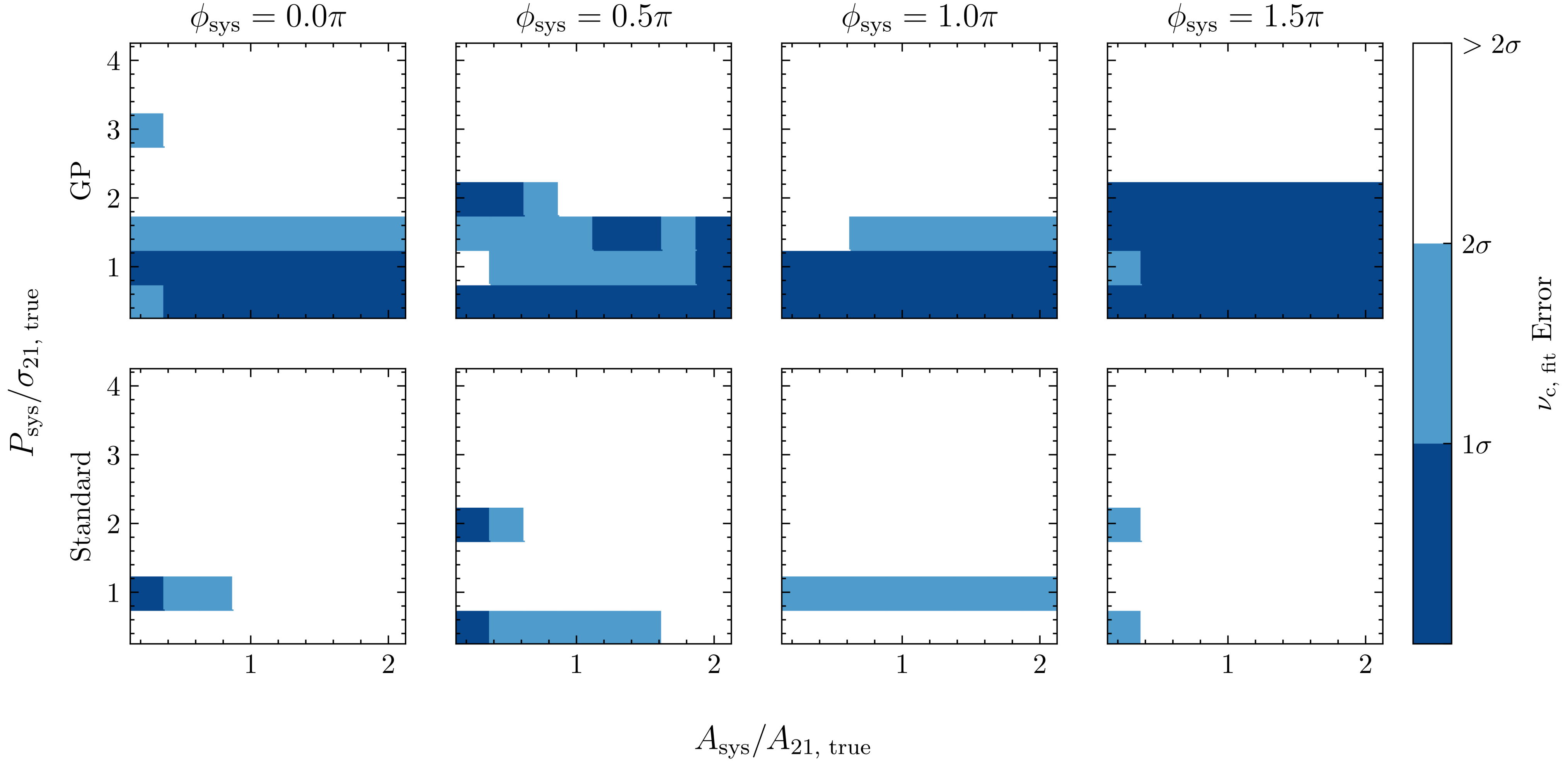
Results – Bayes Factor



Results – Fitted Signal Amplitude



Results – Fitted Centre Frequency



Future – Gaussian Process Regression

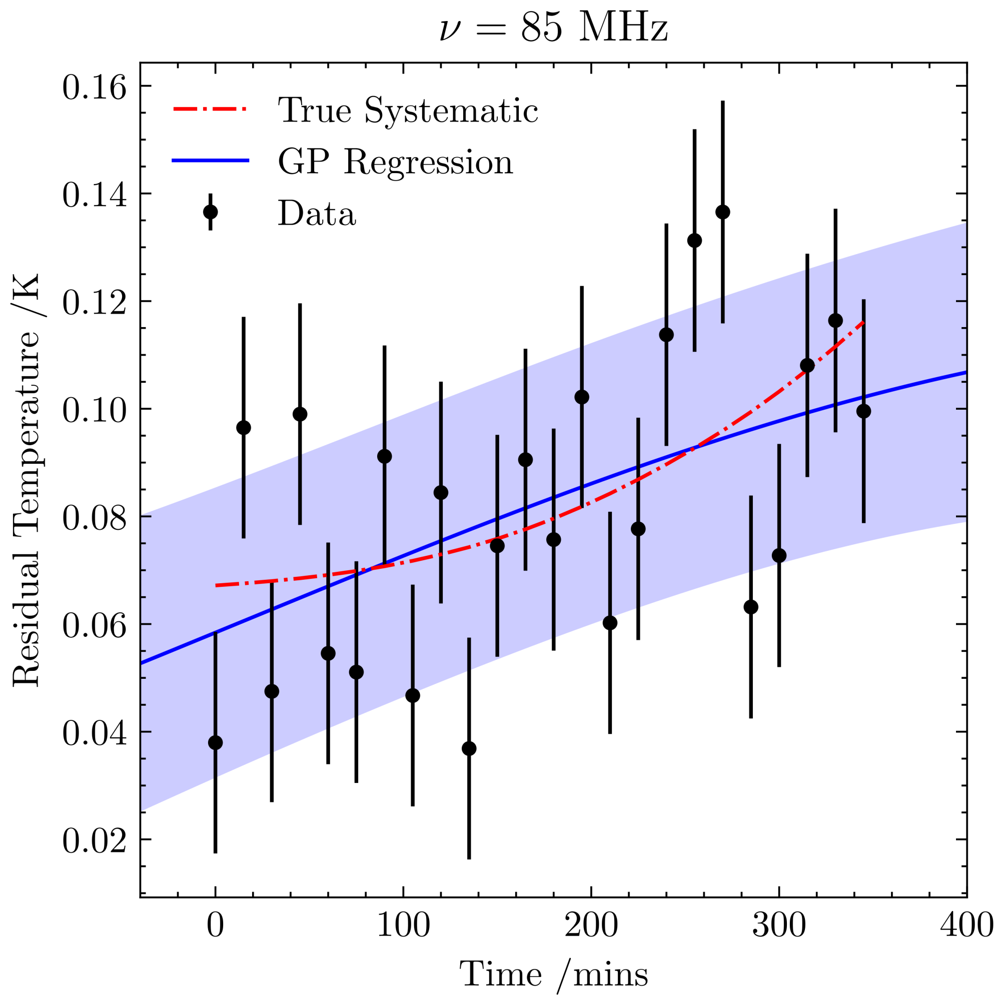
- Can also use Gaussian Processes for regression
- Mean function of the Gaussian Process posterior given by

$$\mu(\mathbf{t}_{\text{pred}}) = K(\mathbf{t}_{\text{pred}}, \mathbf{t}_{\text{data}})K(\mathbf{t}_{\text{data}}, \mathbf{t}_{\text{data}})^{-1}\mathbf{T}_{\text{data}}$$

- Can be used to predict temperature, $\mathbf{T}_{\text{pred}}$, of model residuals at some time $\mathbf{t}_{\text{pred}}$

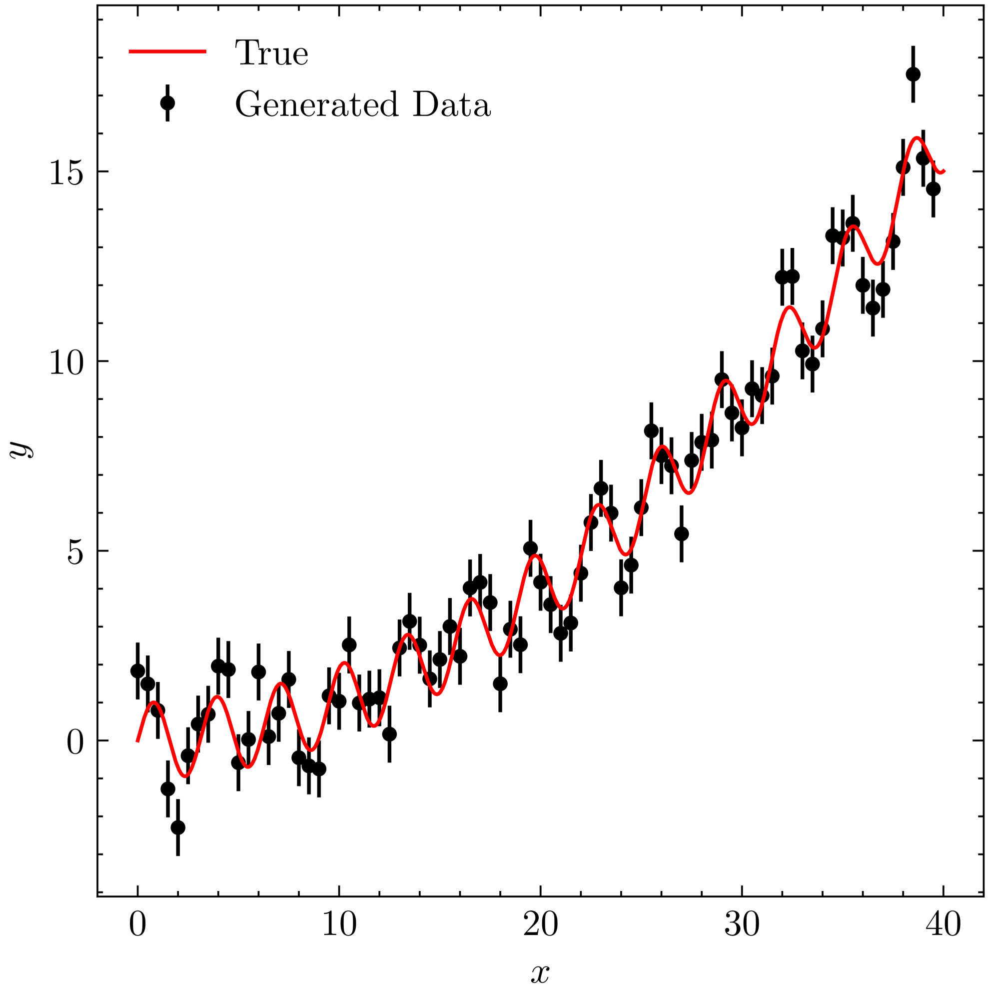
Future – Gaussian Process Regression

- Use weighted mean Gaussian Process hyperparameters to get a smooth fit to model residuals
- Could possibly be used to inform future time-varying systematic models



Future – Automatic Kernel Selection

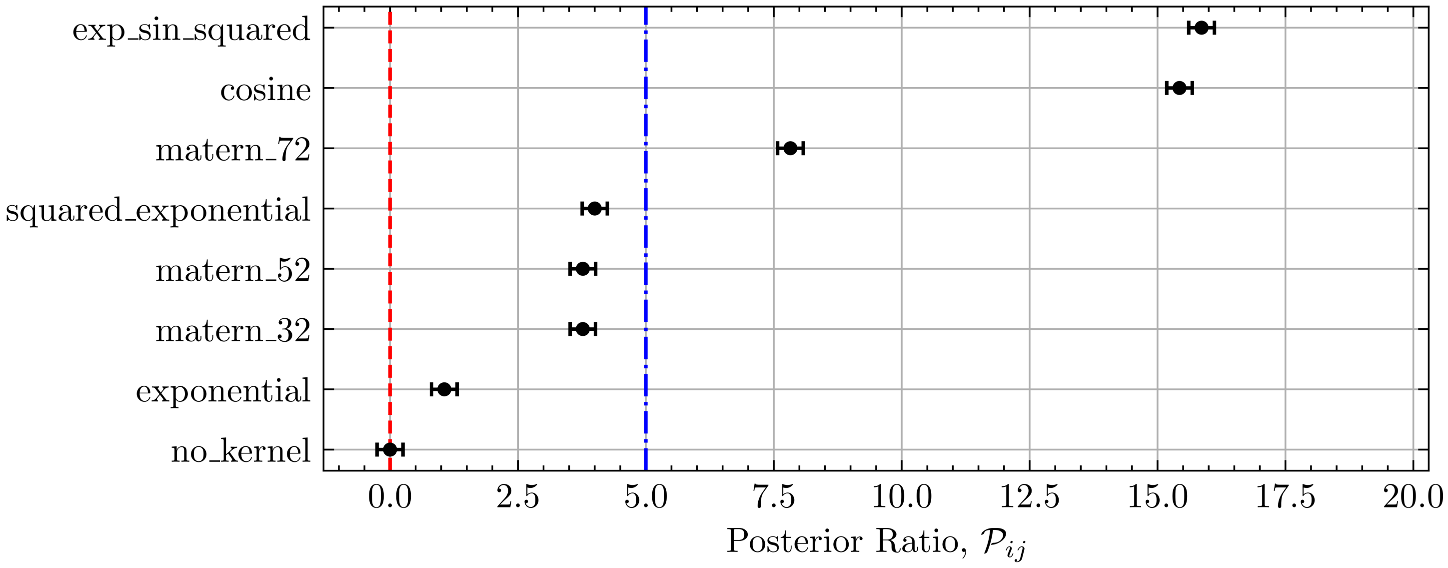
- Kernel choice is arbitrary so can use Bayes factor to inform us (Hee et al. 2015, Kroupa et al. in prep)
- Basic test with a quadratic curve with a sinusoidal residual
- Fit for quadratic but not sinusoid



Future – Automatic Kernel Selection

- Uses PolyChord to sample over all kernels using a choice parameter, c
- Uses the posterior ratio as a proxy for Bayes Factor

$$\mathcal{P}_{ij} = \log \frac{\Pr(c=j|\mathbf{D},\mathbf{M})}{\Pr(c=i|\mathbf{D},\mathbf{M})}$$



Conclusions

- Using Gaussian Processes to account for time correlated residuals improves fitting
- General method – no systematic model required
- Regression can inform future models of systematics
- Automatic Kernel Selection can help select most appropriate kernel