Noise and Singularities in Bayesian Calibration Methods for Global 21-cm Cosmology Experiments

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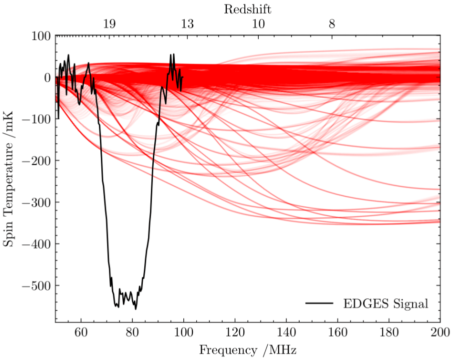
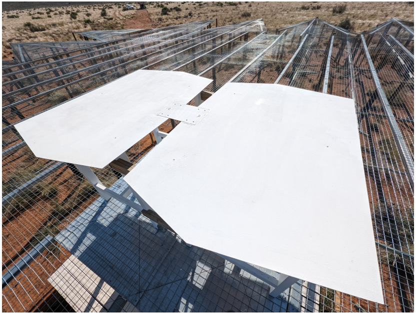
Global 21-cm Cosmology

EDGES (Experiment to Detect the Global EoR Signature)



- Claimed detection of 21-cm signal at 78 MHz (Bowman et al., 2018)
- Unusually deep signal, requiring exotic physics
- Concerns that there is a residual systematic in the data (Hills et al., 2018; Sims and Pober, 2020)

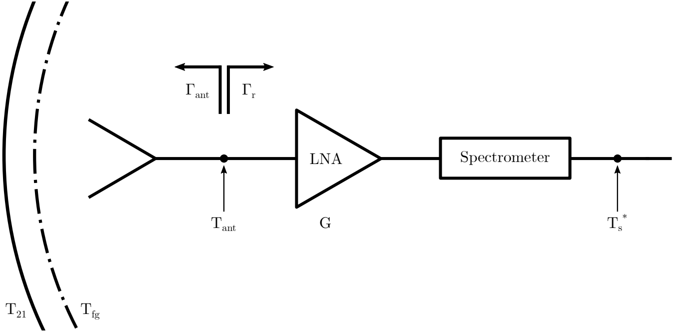
REACH



REACH Antenna

REACH Receiver

Global Experiment Receiver System



- Low Noise Amplifier (LNA) with gain G transforms $T_{\mathrm{ant}} \rightarrow T_{\mathrm{s}}^{*}$

What Is Calibration?

- Calibration relates input voltage (measured by a spectrometer as power spectral density, PSD) to temperature
- In general this relation can be written as

$$P_{\text{source}} = kBgM(T_{\text{source}} + T_{\text{rec}}) \tag{1}$$

- g is the source independent gain of the low noise amplifier (LNA) and M is the impendence mismatch factor

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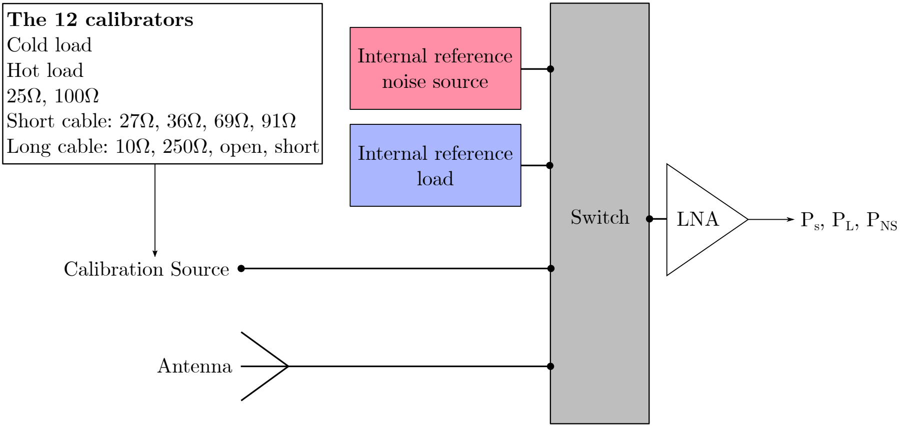
Characterising the LNA with Noise Waves

- When the sources are not impedence matched M and T_{rec} are *source dependent* – simple Dicke switching cannot be used
- One way to characterise the LNA is using 'noise waves' (Meys, 1978; Monsalve et al., 2017)

$$T_{\mathrm{NS}}\left(\frac{P_{\mathrm{s}}-P_{\mathrm{L}}}{P_{\mathrm{NS}}-P_{\mathrm{L}}}\right)+T_{\mathrm{L}}=T_{\mathrm{s}}\left[\frac{1-|\Gamma_{\mathrm{s}}|^2}{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}|^2}\right]+T_{\mathrm{unc}}\left[\frac{|\Gamma_{\mathrm{s}}|^2}{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}|^2}\right]$$
$$+T_{\cos}\left[\frac{\mathrm{Re}\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}}\right)}{\sqrt{1-|\Gamma_{\mathrm{r}}|^2}}\right]+T_{\sin}\left[\frac{\mathrm{Im}\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}}\right)}{\sqrt{1-|\Gamma_{\mathrm{r}}|^2}}\right] \tag{2}$$

- "Calibration equation" - three noise wave parameters: T_{unc}, $T_{\cos}$, $T_{\sin}$
- We also fit for T_{NS}, T_{L} (Roque et al., 2021)

REACH Receiver



Exploiting the Linearity

- All measured quantities - PSDs, reflection coefficients absorbed into X terms to give

$$T_s(\nu) = X_{\text{unc}}\, T_{\text{unc}} + X_{\cos}\, T_{\cos} + X_{\sin}\, T_{\sin} + X_{\text{NS}}\, T_{\text{NS}} + X_{\text{L}}\, T_{\text{L}} \tag{3}$$

- Can write this as a linear equation

$$\mathbf{T}_{\text{s}} = \mathbf{X}\boldsymbol{\Theta} + \sigma \tag{4}$$

- *measured* – $\mathbf{X} = \begin{pmatrix} X_{\text{unc}} & X_{\cos} & X_{\sin} & X_{\text{NS}} & X_{\text{L}} \end{pmatrix}$
  fitted for – $\boldsymbol{\Theta} = \begin{pmatrix} T_{\text{unc}} & T_{\cos} & T_{\sin} & T_{\text{NS}} & T_{\text{L}} \end{pmatrix}^T$

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Calibration Likelihood

- For all methods in this talk we will use a Gaussian Likelihood

$$\mathcal{L} = \frac{1}{\sqrt{|2\pi\mathbf{C}|}} \exp\left\{-\frac{1}{2}(\mathbf{T} - \mathbf{X\Theta})^T \mathbf{C}^{-1} (\mathbf{T} - \mathbf{X\Theta})\right\} \tag{5}$$

- We fit polynomials to the noise wave parameters, where Θ are the coefficients
- Use this and prior, $\pi(\mathbf{\Theta})$, to sample the posterior distribution, $\mathcal{P} = \mathcal{L}\pi/\mathcal{Z}$
- Calibration solution is the weighted mean of posterior

Conjugate Priors Method

- Method introduced in Roque et al., 2021
- Choose prior to be a Normal Inverse Gamma distribution, N-Γ$(\mu_{\mathbf{\Theta}}, \mathbf{V}_{\mathbf{\Theta}}, a, b)$
- Posterior is then calculated analytically to be the same distribution, N-Γ$(\mu^*, \mathbf{V}^*, a^*, b^*)$
- Transformations of $\mu_{\mathbf{\Theta}} \rightarrow \mu^*$ etc. can be calculated analytically with data to quickly evaluate the posterior
- Requires data covariance to be $\mathbf{C} = \frac{1}{\sigma^2}\mathbf{I}$

Estimating the Noise

- Need a figure of merit for calibration
- Assumes PSD noise is Gaussian and S_{11} measurements are noiseless
- Propagate PSD noise through the noise wave parameter equations
- Key points:

$$\sigma_{T_s} \propto 1/(1 - |\Gamma_{\mathrm{s}}|^2) \tag{6}$$

$$\sigma_{T_s} \propto T_{\mathrm{NS}}^{\mathrm{fit}} \tag{7}$$

Drawbacks of Conjugate Priors

- Choice of conjugate prior assumes all source temperatures have same noise, σ
  - As seen $\sigma \propto 1/(1 - |\Gamma_s|^2)$, where $|\Gamma_s| \in (0, 1)$
- Gradient descent can get stuck in local minima

Γ-Weighted Conjugate Priors

- Rewrite the calibration equation as

$$\mathbf{T}'_{\mathrm{s}} = (1 - |\Gamma_{\mathrm{s}}|^2)\mathbf{T}_{\mathrm{s}} = (1 - |\Gamma_{\mathrm{s}}|^2)\mathbf{X}\boldsymbol{\Theta} = \mathbf{X}'\boldsymbol{\Theta} \tag{8}$$

- As a result, $\sigma' \not\propto 1/(1 - |\Gamma_{\mathrm{s}}|^2)$
- Physical motivation for EDGES' down-weighting of cables in Murray et al., 2022

Marginalised Polynomial Method

- Fit for calibrator noise parameters separately

$$\eta = \begin{pmatrix} \sigma_0 & \sigma_1 & \dots \end{pmatrix} \tag{9}$$

- We can also fit for the polynomial orders as parameters

$$\mathbf{n} = \begin{pmatrix} n_{\mathrm{unc}} & n_{\cos} & n_{\sin} & n_{\mathrm{NS}} & n_{\mathrm{L}} \end{pmatrix} \tag{10}$$

- We then marginalise over $\mathbf{\Theta}$ giving the marginal likelihood

$$\begin{aligned} \log \mathcal{L}(\eta, \mathbf{n}) =& \frac{1}{2} \log \left| \frac{\mathbf{\Sigma}_P}{\mathbf{\Sigma}_\pi} \right| - \frac{1}{2} (\mu_P - \mu_\pi)^T \mathbf{\Sigma}_\pi^{-1} (\mu_P - \mu_\pi) \\ & - \frac{1}{2} \log |2\pi \mathbf{C}| - \frac{1}{2} (\mathbf{T} - \mathbf{X}\mu_P)^T \mathbf{C}^{-1} (\mathbf{T} - \mathbf{X}\mu_P), \end{aligned} \tag{11}$$

Marginalised Polynomial Method (cont.)

- Now we numerically sample over noise parameters and the polynomial orders
- Polynomial coefficients are then sampled from

$$\mathbf{\Theta} \sim \mathcal{N}(\mu, \mathbf{\Sigma}). \tag{12}$$

- Much more flexible method which allows the use of arbitrary noise models

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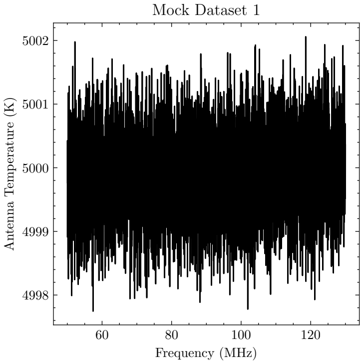
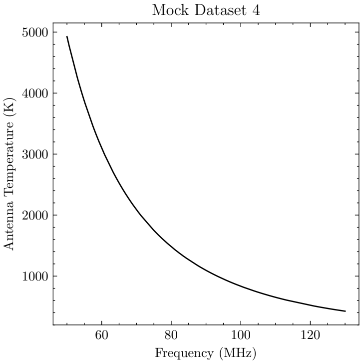
Mock Data

- Full simulation of REACH

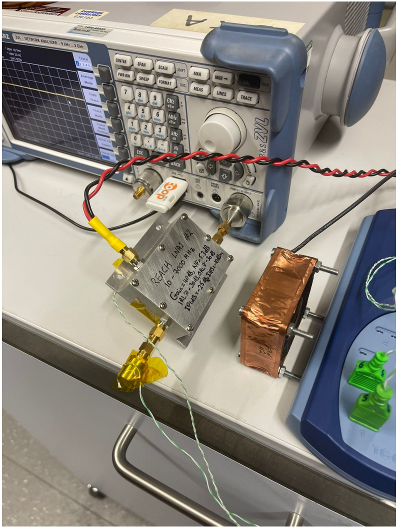


Flat Spectrum



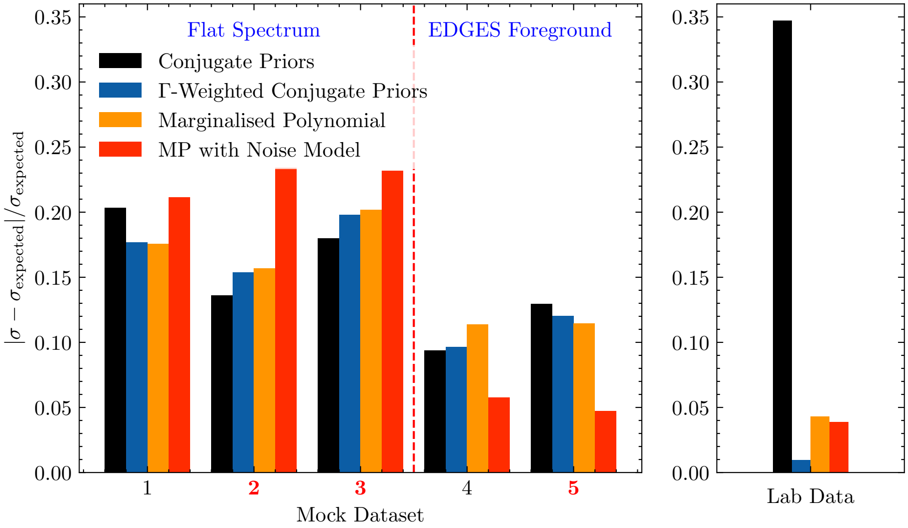
EDGES Foreground

Lab Data

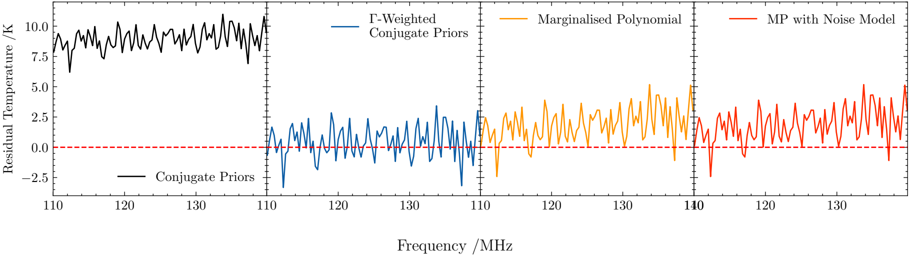


Lab Data Setup

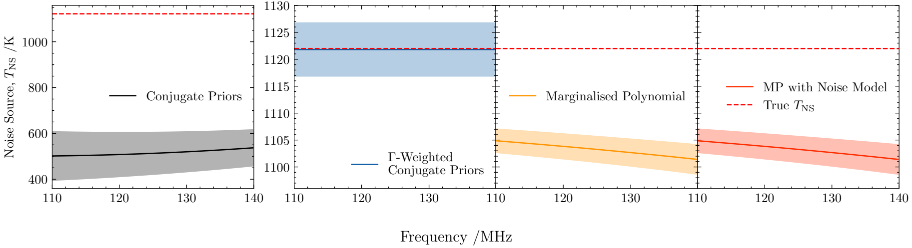
Method Comparison (RMSE)



Method Comparison (Residuals)



Method Comparison (T_{NS} Recovery)



Summary

- In order to determine absolute temperatures and remove receiver systematics we must calibrate – noise waves provide a framework that we can use to characterise the LNA
- We can modify the conjugate priors method to mitigate the biases introduced by singular equations and varying noise
- Achieving the lowest RMSE is not the ultimate goal of calibration – the theoretical noise floor must be considered

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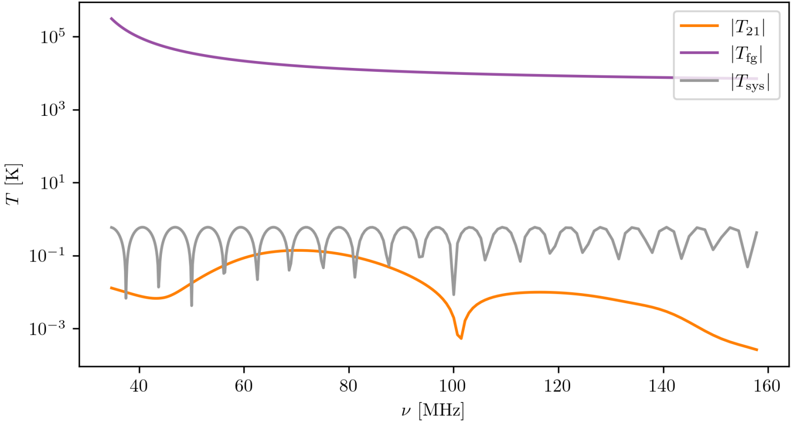
Dynamic Range Problem



Credit: Harry Bevins

Polynomial Order Posterior