A Marginalised Bayesian Noise Wave Calibration Method

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Characterising the LNA with Noise Waves

- Noise Wave formalism models the LNA response (Meys, 1978; Monsalve et al., 2017)

$$T_{\mathrm{NS}}\left(\frac{P_{\mathrm{s}}-P_{\mathrm{L}}}{P_{\mathrm{NS}}-P_{\mathrm{L}}}\right)+T_{\mathrm{L}}=T_{\mathrm{s}}\left[\frac{1-|\Gamma_{\mathrm{s}}|^2}{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}|^2}\right]+T_{\mathrm{unc}}\left[\frac{|\Gamma_{\mathrm{s}}|^2}{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}|^2}\right] + T_{\mathrm{cos}}\left[\frac{\mathrm{Re}\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}}\right)}{\sqrt{1-|\Gamma_{\mathrm{r}}|^2}}\right]+T_{\mathrm{sin}}\left[\frac{\mathrm{Im}\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}}\right)}{\sqrt{1-|\Gamma_{\mathrm{r}}|^2}}\right] \tag{1}$$

- "Calibration equation" - three noise wave parameters: T_{unc}, T_{cos}, T_{sin}
- We also fit for T_{NS}, T_{L} (Roque et al., 2021)

Drawbacks of Conjugate Priors

- Conjugate priors calibration method (Roque et al., 2021) uses a Normal Inverse Gamma prior so the posterior can be evaluated analytically, quickly
- But choice of conjugate prior assumes all calibrator source temperatures have same noise, σ
  - For noise wave parameters $\sigma \propto 1/(1 - |\Gamma_s|^2)$, where $|\Gamma_s| \in (0, 1)$
- Polynomial order selection - gradient descent can get stuck in local minima

Marginalised Polynomial Method

- We can fit for the polynomial orders as parameter

$$\mathbf{n} = \begin{pmatrix} n_{\text{unc}} & n_{\cos} & n_{\sin} & n_{\text{NS}} & n_{\text{L}} \end{pmatrix} \tag{2}$$

- Fit for calibrator noise parameters

$$\eta = \begin{pmatrix} \sigma_0 & \sigma_1 & \dots \end{pmatrix} \tag{3}$$

- To speed up the fit we marginalise over $\mathbf{\Theta}$
- Marginal likelihood is then

$$\mathcal{L}_{\text{eff}}(\eta, \mathbf{n}) = \sqrt{\frac{1}{|2\pi\mathbf{C}||\mathbf{\Sigma}_p||\mathbf{\Sigma}^{-1}|}} \exp\left\{\frac{1}{2}\mu^T\mathbf{\Sigma}^{-1}\mu - \frac{1}{2}\mathbf{T}^T\mathbf{C}^{-1}\mathbf{T}\right\} \tag{4}$$

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Method Comparison (RMSE)