Modelling Drifts in Receiver Calibrations for Global 21-cm Cosmology Experiments

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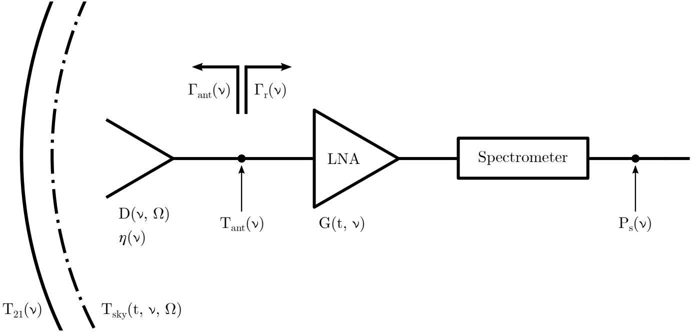
Global Experiment Receiver System



- Low Noise Amplifier (LNA) with gain G outputs amplified power

What Is Calibration?

- Calibration relates output power spectral density (PSD) to temperature
- In general this relation can be written as

$$P_{\text{source}} = kBgM(T_{\text{source}} + T_{\text{rec}}) \tag{1}$$

- g is the source independent gain of the low noise amplifier (LNA) and M is the impedance mismatch factor
- T_{rec} is added noise temperature from LNA

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Characterising the LNA with Noise Waves

- When the sources are not impedance matched M and T_{rec} are *source dependent* – simple Dicke switching cannot be used
- One way to characterise the LNA is using 'noise waves' (Meys, 1978; Monsalve et al., 2017)

$$T'_{\mathrm{NS}}\left(\frac{P_{\mathrm{s}}-P_{\mathrm{L}}}{P_{\mathrm{NS}}-P_{\mathrm{L}}}\right)+T'_{\mathrm{L}}=T_{\mathrm{s}}\left[\frac{1-|\Gamma_{\mathrm{s}}|^2}{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}|^2}\right]+T_{\mathrm{unc}}\left[\frac{|\Gamma_{\mathrm{s}}|^2}{|1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}|^2}\right]$$
$$+T_{\cos}\left[\frac{\mathrm{Re}\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}}\right)}{\sqrt{1-|\Gamma_{\mathrm{r}}|^2}}\right]+T_{\sin}\left[\frac{\mathrm{Im}\left(\frac{\Gamma_{\mathrm{s}}}{1-\Gamma_{\mathrm{s}}\Gamma_{\mathrm{r}}}\right)}{\sqrt{1-|\Gamma_{\mathrm{r}}|^2}}\right] \tag{2}$$

- "Calibration equation" - three noise wave parameters: T_{unc}, $T_{\cos}$, $T_{\sin}$
- We also fit for T'_{NS}, T'_{L} (Roque et al., 2021)

Degeneracies in the Noise Wave Parameters

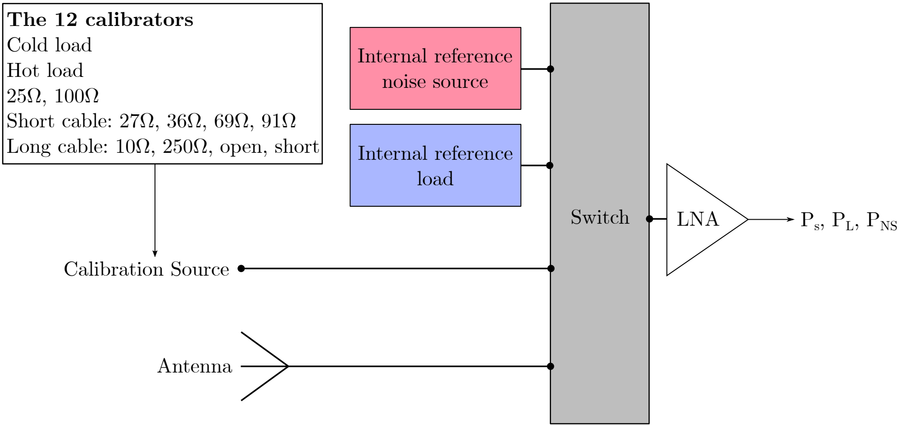
- Fitting for T'_{NS} and T'_{L} fits for the mismatch in reflection coefficients, however introduces degeneracies

$$
\begin{aligned}
T'_{\mathrm{NS}} = \frac{1}{1-|\Gamma_{\mathrm{rec}}|^2} \Big[& T_{\mathrm{NS}}\left(1-\left|\Gamma_{\mathrm{NS}}\right|^2\right)|F_{\mathrm{NS}}|^2 - T_{\mathrm{L}}\left(1-\left|\Gamma_{\mathrm{L}}\right|^2\right)|F_{\mathrm{L}}|^2 \\
& + T_{\mathrm{unc}}\left(\left|\Gamma_{\mathrm{NS}}\right|^2|F_{\mathrm{NS}}|^2 - \left|\Gamma_{\mathrm{L}}\right|^2|F_{\mathrm{L}}|^2\right) \\
& + T_{\mathrm{cos}}\left(\left|\Gamma_{\mathrm{NS}}\right||F_{\mathrm{NS}}|\cos\alpha_{\mathrm{NS}} - \left|\Gamma_{\mathrm{L}}\right||F_{\mathrm{L}}|\cos\alpha_{\mathrm{L}}\right) \\
& + T_{\mathrm{sin}}\left(\left|\Gamma_{\mathrm{NS}}\right||F_{\mathrm{NS}}|\sin\alpha_{\mathrm{NS}} - \left|\Gamma_{\mathrm{L}}\right||F_{\mathrm{L}}|\sin\alpha_{\mathrm{L}}\right)\Big]
\end{aligned} \tag{3}
$$

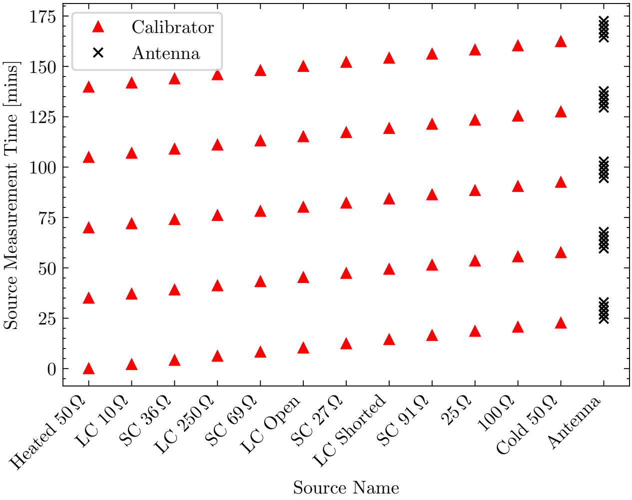
and

$$
\begin{aligned}
T'_{\mathrm{L}} = \frac{1}{1-|\Gamma_{\mathrm{rec}}|^2} \Big[& T_{\mathrm{L}}\left(1-\left|\Gamma_{\mathrm{L}}\right|^2\right)|F_{\mathrm{L}}|^2 + T_{\mathrm{unc}}\left|\Gamma_{\mathrm{L}}\right|^2|F_{\mathrm{L}}|^2 \\
& + T_{\mathrm{cos}}\left|\Gamma_{\mathrm{L}}\right||F_{\mathrm{L}}|\cos\alpha_{\mathrm{L}} + T_{\mathrm{sin}}\left|\Gamma_{\mathrm{L}}\right||F_{\mathrm{L}}|\sin\alpha_{\mathrm{L}}\Big]
\end{aligned} \tag{4}
$$

REACH Receiver



REACH Observations



Exploiting the Linearity

- All measured quantities – PSDs, reflection coefficients – absorbed into X terms to give

$$T_s(\nu) = X_{\text{unc}}\,T_{\text{unc}} + X_{\text{cos}}\,T_{\text{cos}} + X_{\text{sin}}\,T_{\text{sin}} + X_{\text{NS}}\,T_{\text{NS}} + X_{\text{L}}\,T_{\text{L}} \tag{5}$$

- Can write this as a linear equation

$$\mathbf{T}_{\text{s}} = \mathbf{X}\boldsymbol{\Theta} + \sigma \tag{6}$$

- *measured* – $\mathbf{X} = \begin{pmatrix} X_{\text{unc}} & X_{\text{cos}} & X_{\text{sin}} & X_{\text{NS}} & X_{\text{L}} \end{pmatrix}$
  fitted for – $\boldsymbol{\Theta} = \begin{pmatrix} T_{\text{unc}} & T_{\text{cos}} & T_{\text{sin}} & T_{\text{NS}} & T_{\text{L}} \end{pmatrix}^T$

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Calibration Likelihood

- For this talk we will use a Gaussian Likelihood

$$\mathcal{L} = \frac{1}{\sqrt{|2\pi\mathbf{C}|}} \exp\left\{ -\frac{1}{2}(\mathbf{T} - \mathbf{X\Theta})^T \mathbf{C}^{-1} (\mathbf{T} - \mathbf{X\Theta}) \right\} \tag{7}$$

- Where $\mathbf{\Theta}$ are the model coefficients
- Sample the posterior distribution analytically using the conjugate priors method (Roque et al., 2021)
- Calibration solution is the weighted mean of posterior

Surface Fitting

- We fit a polynomial frequency-time surface

$$T_{\mathrm{NWP}}(\nu, t) = \sum_i \sum_j A_{ij} \cdot \nu^i t^j \tag{8}$$

- Which can be written linearly as

$$\mathbf{T}_i = \mathbf{X}_i \mathbf{B}_i \mathbf{\Theta} \tag{9}$$

where

$$\mathbf{\Theta} = \begin{pmatrix} A_{00} \\ A_{01} \\ A_{10} \\ A_{11} \\ A_{02} \\ \vdots \end{pmatrix}, \mathbf{B}_i = \begin{pmatrix} 1 & t_i & \nu_0 & \nu_0 t_i & t_i^2 & \dots \\ 1 & t_i & \nu_1 & \nu_1 t_i & t_i^2 & \dots \\ 1 & t_i & \nu_2 & \nu_2 t_i & t_i^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{10}$$

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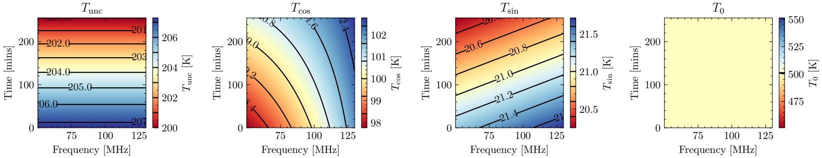
Data Simulation



$$P_s = kBg\big(T_s(1-|\Gamma_s|^2)|F_s|^2 + T_{\mathrm{unc}}|\Gamma_s|^2|F_s|^2 + T_{\cos}\mathrm{Re}(\Gamma_s F_s) + T_{\sin}\mathrm{Im}(\Gamma_s F_s) + T_0\big) \tag{11}$$

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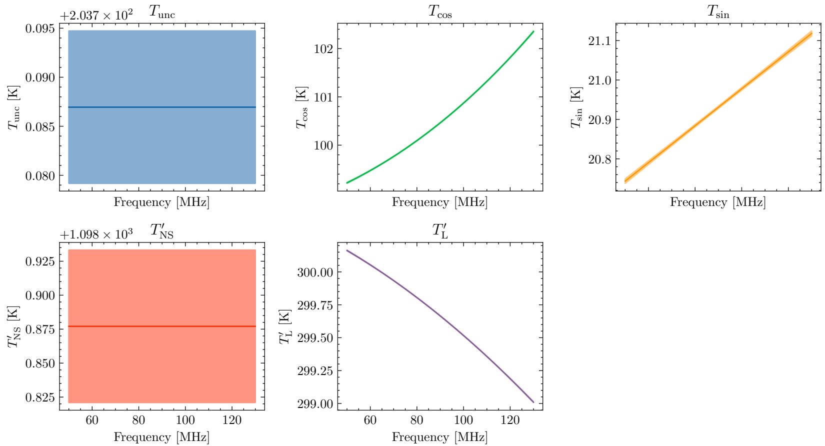
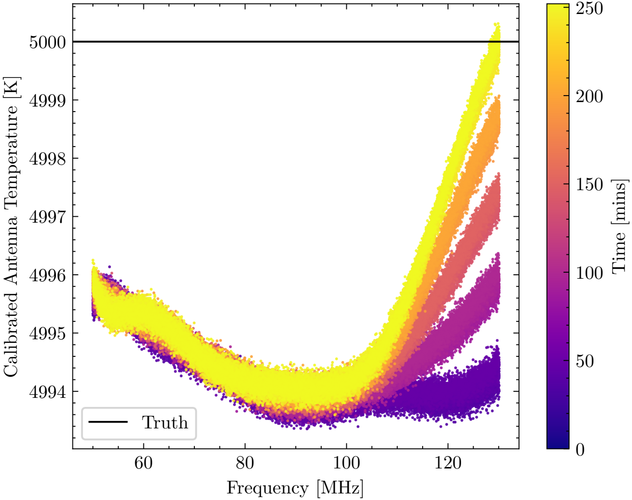
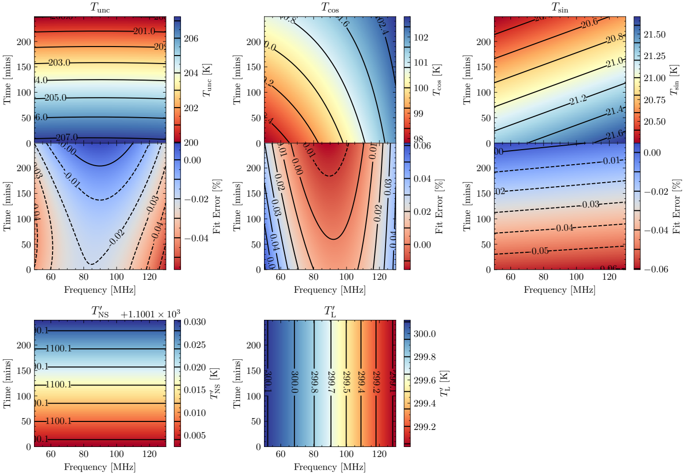
1-D Conjugate Priors



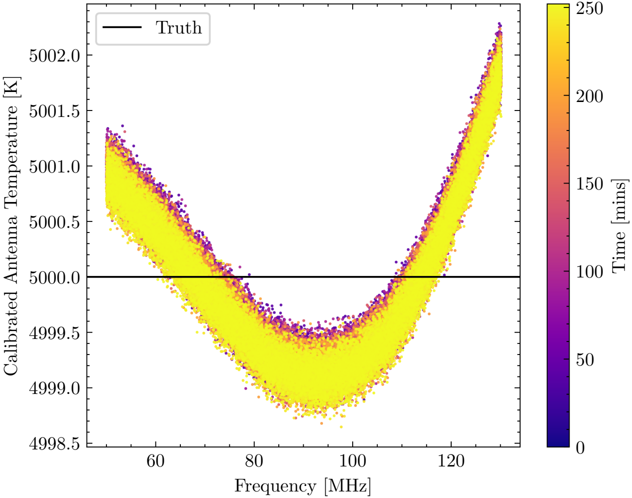
1-D Conjugate Priors



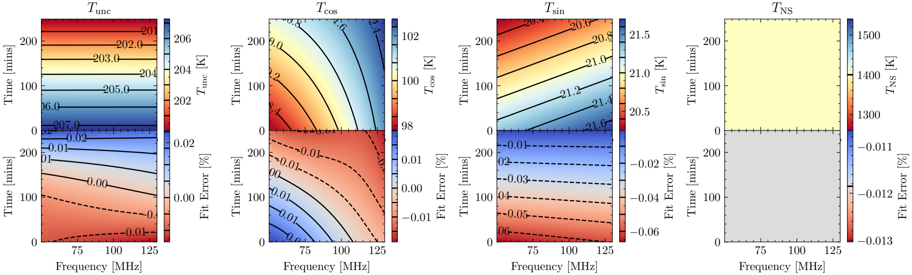
Surface Fitting



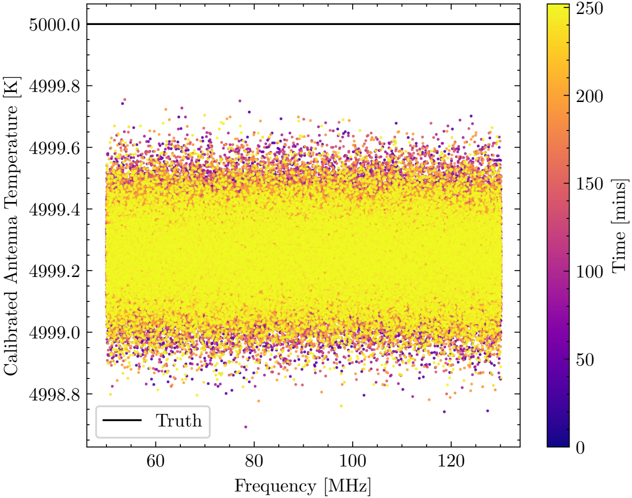
Surface Fitting



Surface Fitting Including Γ_L and Γ_{NS} Terms



Surface Fitting Including Γ_L and Γ_{NS} Terms



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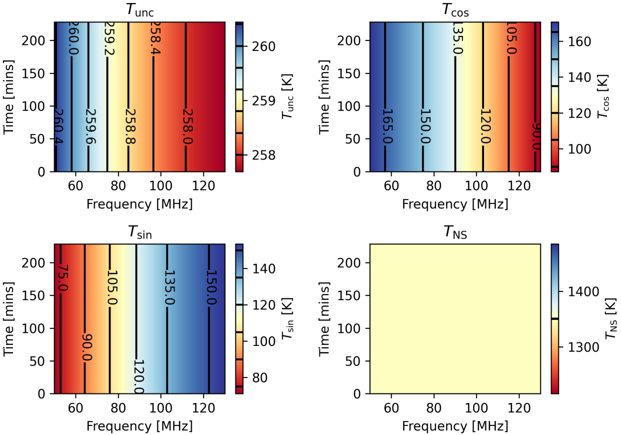
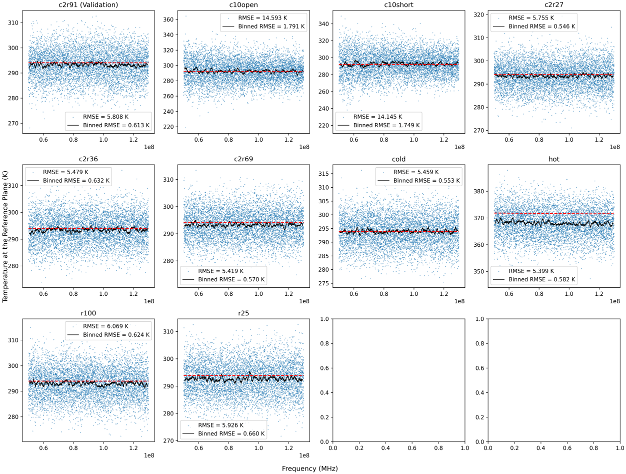
Fitting REACH Data



Fitting REACH Data



Summary

- If the state of the receiver is drifting over the course of an observation it is necessary to consider this time change
- Fitting a frequency-time polynomial surface to simulated data removes drifts in the calibrated antenna solution but a leaves a significant chromatic residual
- Mitigating degeneracies by explicitly including Γ_{L} and Γ_{NS} terms removes this chromatic residual
- Tests on data from the REACH instrument show that the REACH receiver is not drifting over an observation

Extra: Calibration Equation with Γ_{NS} and Γ_{L} Terms

$$T_s(\nu) = \bar{X}_{\mathrm{unc}}\, T_{\mathrm{unc}} + \bar{X}_{\mathrm{cos}}\, T_{\mathrm{cos}} + \bar{X}_{\mathrm{sin}}\, T_{\mathrm{sin}} + \bar{X}_{\mathrm{NS}}\, T_{\mathrm{NS}} + \bar{X}_{\mathrm{L}}\, T_{\mathrm{L}}\,, \tag{12}$$

where

$$\bar{X}_{\mathrm{NS}} = \frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{P_{\mathrm{NS}} - P_{\mathrm{L}}} \frac{(1 - |\Gamma_{\mathrm{NS}}|^2)|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)|1 - \Gamma_{\mathrm{NS}}\Gamma_{\mathrm{rec}}|^2}\,, \tag{13}$$

$$\bar{X}_{\mathrm{L}} = \left[1 - \frac{P_{\mathrm{s}} - P_{\mathrm{L}}}{P_{\mathrm{NS}} - P_{\mathrm{L}}}\right] \frac{(1 - |\Gamma_{\mathrm{L}}|^2)|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{(1 - |\Gamma_{\mathrm{s}}|^2)|1 - \Gamma_{\mathrm{L}}\Gamma_{\mathrm{rec}}|^2}\,, \tag{14}$$

$$\bar{X}_{\mathrm{unc}} = \bar{X}_{\mathrm{NS}} \frac{|\Gamma_{\mathrm{NS}}|^2}{1 - |\Gamma_{\mathrm{NS}}|^2} + \bar{X}_{\mathrm{L}} \frac{|\Gamma_{\mathrm{L}}|^2}{1 - |\Gamma_{\mathrm{L}}|^2} - \frac{|\Gamma_{\mathrm{s}}|^2}{1 - |\Gamma_{\mathrm{s}}|^2}\,, \tag{15}$$

$$\begin{aligned}\bar{X}_{\mathrm{cos}} = \bar{X}_{\mathrm{NS}} \cdot \mathrm{Re}\left(\frac{\Gamma_{\mathrm{NS}}}{1 - \Gamma_{\mathrm{NS}}\Gamma_{\mathrm{rec}}} \cdot \frac{|1 - \Gamma_{\mathrm{NS}}\Gamma_{\mathrm{rec}}|^2}{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}(1 - |\Gamma_{\mathrm{NS}}|^2)}\right) + \bar{X}_{\mathrm{L}} \cdot \mathrm{Re}\left(\frac{\Gamma_{\mathrm{L}}}{1 - \Gamma_{\mathrm{L}}\Gamma_{\mathrm{rec}}} \cdot \frac{|1 - \Gamma_{\mathrm{L}}\Gamma_{\mathrm{rec}}|^2}{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}(1 - |\Gamma_{\mathrm{L}}|^2)}\right) \\ - \mathrm{Re}\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \cdot \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}(1 - |\Gamma_{\mathrm{s}}|^2)}\right)\,,\end{aligned} \tag{16}$$

$$\begin{aligned}\bar{X}_{\mathrm{sin}} = \bar{X}_{\mathrm{NS}} \cdot \mathrm{Im}\left(\frac{\Gamma_{\mathrm{NS}}}{1 - \Gamma_{\mathrm{NS}}\Gamma_{\mathrm{rec}}} \cdot \frac{|1 - \Gamma_{\mathrm{NS}}\Gamma_{\mathrm{rec}}|^2}{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}(1 - |\Gamma_{\mathrm{NS}}|^2)}\right) + \bar{X}_{\mathrm{L}} \cdot \mathrm{Im}\left(\frac{\Gamma_{\mathrm{L}}}{1 - \Gamma_{\mathrm{L}}\Gamma_{\mathrm{rec}}} \cdot \frac{|1 - \Gamma_{\mathrm{L}}\Gamma_{\mathrm{rec}}|^2}{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}(1 - |\Gamma_{\mathrm{L}}|^2)}\right) \\ - \mathrm{Im}\left(\frac{\Gamma_{\mathrm{s}}}{1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}} \cdot \frac{|1 - \Gamma_{\mathrm{s}}\Gamma_{\mathrm{rec}}|^2}{\sqrt{1 - |\Gamma_{\mathrm{rec}}|^2}(1 - |\Gamma_{\mathrm{s}}|^2)}\right)\,.\end{aligned} \tag{17}$$

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